Assignment 1

Coverage: 15.1 in Text.

Exercises: 15.1. No 7, 9, 11, 16, 18, 20, 25, 27, 32, 34.

Submit no. 20, 32, and 34 by Jan 19.

Supplementary Problems

1. Consider the function $\varphi(x) = x^{-a}$ where a is positive for $x \in (0, 1]$ and set $\varphi(0) = 1$ so that φ is a well-defined function on $[0, 1]$. Show that φ is not integrable on $[0, 1]$. This is the simplest example of an unbounded function. Suggestion: You could use proof by contradiction. Assume it is integrable and then draw a contradiction.

2. Consider the function H in $\mathbb{R}^2$ defined by $H(x, y) = 1$ whenever x, y are rational numbers and equals to 0 otherwise. Show that H is not integrable in any rectangle.

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2020B (Spring Term, 2021)
Advanced Calculus II
Assignment 1

Please hand in the following questions by 19th Jan 23:00.

Q20

$$\iint_R xye^{xy^2}\,dA, \qquad R:\ 0 \le x \le 2,\ 0 \le y \le 1$$

Q32

Evaluate

$$\int_{-1}^{1}\int_{0}^{\pi/2} x \sin\sqrt{y}\,dy\,dx$$

Q34

Use Fubini's theorem to evaluate

$$\int_0^1 \int_0^3 xe^{xy}\,dx\,dy$$